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# Adaptive Attitude Control for an Artificial Satellite with Mobile Bodies

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**Feedforward control with a parameter estimation law for an artificial satellite with antennas or manipulators is presented. The proposed method utilizes the linearity of the unknown parameters on the angular momentum. It is possible to append an estimation capability by adding a few terms to conventional feedforward control. A compensation method for the time delay in the velocity control loop of the reaction wheels is also proposed, and its effects on the parameter estimation results are analyzed. The analytical results coincide well with simulation results, and the effects of the time delay are expressed quantitatively.**

## Nomenclature

$A(\phi, \dot{\phi}) \in R^{3\times m}$ = matrix whose elements are products of $\dot{\phi}$ and a polynomial of trigonometric functions of $\phi$
$f(\phi, \dot{\phi}) \in R^{3\times 1}$ = angular momentum of mobile bodies around center of mass of whole satellite
$\boldsymbol{h} \in R^{3\times 1}$ = angular momentum of reaction wheels
$\boldsymbol{h}_c \in R^{3\times 1}$ = commanded values of $\boldsymbol{h}$
$i_{ax}, i_{ay}, i_{az}$ = moments of inertia of antenna; see numerical example
$i_{bx}, i_{by}, i_{bz}$ = moments of inertia of satellite main body; see numerical example
$\boldsymbol{K}_I(>0) \in R^{3\times 3}$ = integral gain for attitude control
$\boldsymbol{K}_P(>0) \in R^{3\times 3}$ = proportional gain for attitude control
$\boldsymbol{M}(\phi) \in R^{3\times 3}$ = inertia matrix of whole satellite around its center of mass
$m$ = number of components of $\alpha$
$m_a, m_b$ = mass of antenna and satellite main body; see numerical example
$n$ = degrees of freedom of mobile bodies
$\boldsymbol{P}(>0) \in R^{m\times m}$ = estimation gain
$r_{1x}, r_{1y}, r_{1z}$ = vector components from mass center of satellite main body to antenna attachment point; see numerical example
$r_2$ = distance from attachment point to $\phi_2$ axis (in $z$ direction); see numerical example
$r_3$ = distance from $\phi_2$ axis to antenna mass center (in $z$ direction); see numerical example
$\alpha \in R^{m\times 1}$ = unknown parameters
$\hat{\alpha} \in R^{m\times 1}$ = estimated values of $\alpha$
$\hat{\alpha}_0 \in R^{m\times 1}$ = initial values of $\hat{\alpha}$
$\hat{\alpha}_\infty \in R^{m\times 1}$ = stationary values of $\hat{\alpha}$
$\boldsymbol{\theta} \in R^{3\times 1}$ = small attitude angles of satellite
$\phi \in R^{n\times 1}$ = rotational angles of mobile bodies
$\phi_1, \phi_2$ = antenna angles, $\phi$ is equal to $[\phi_1 \;\; \phi_2]^T$; see numerical example
= time derivative operated on each component of the vectors and matrices

## Introduction

MORE and more artificial satellites have mobile bodies such as antennas and manipulators that move in a wide range. An example is shown in Fig. 1 (Ref. 1). It is the Communications and Broadcasting Engineering Test Satellite (COMETS) under development by National Space Development Agency of Japan (NASDA) and has a mobile antenna of diameter 3.6 m for interorbit communications. The movement of such mobile bodies disturbs the satellite attitude.

Conventionally, the antenna inertia is sensibly lower than the inertia of the satellite main body and it does not induce significant motion. Designing the antenna control and the body control separately has been sufficient.[2] Recently, however, the size and weight of the antennas of the satellites have increased (e.g., COMETS, see Fig. 1) and at the same time they require very precise pointing accuracy.[3,4] Hence, design of the control system must consider the coupling between the main body and the mobile bodies. This paper discusses such a control problem.

Usually feedforward control based on angular momentum[4–7] or torque[8,9] compensation is applied to such cases. The mass property of the large antennas or the manipulators is difficult to measure or predict on the ground. Such model errors in the mass property will lead to the disturbances against the satellite attitude.

To decrease the model error effects, this paper presents feedforward control of the mobile bodies movement with a parameter estimation law. The proposed method is based on the adaptive control law for manipulators on the ground,[10,11] which utilizes the fact that the equations of motion can be expressed linearly in terms of the unknown parameters such as mass and moments of inertia. This linearity also holds for the satellite dynamics, and the proposed method has a simple structure from this linearity, which is the reason for selecting this type of adaptive control. In addition, it uses the angular momentum conservation instead of the equation of motion. Therefore, the control law is simplified. It is possible to append the estimation capability by just adding a few terms to the conventional feedforward control law.

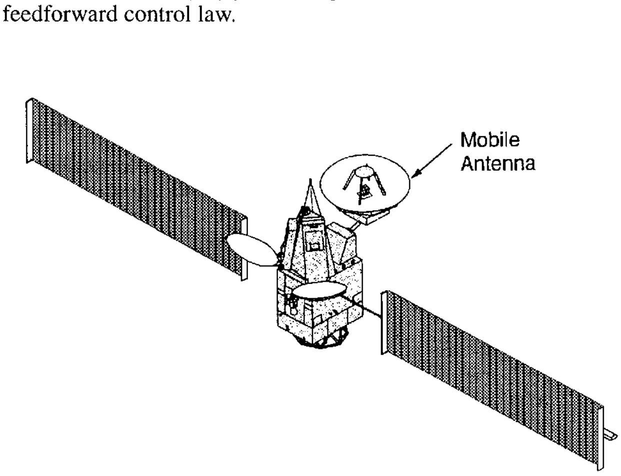


**Fig. 1 Spacecraft with a mobile antenna (from NASDA[1]).**

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When the control law is based on the angular momentum conservation, however, the time delay in the velocity control of the wheels causes some difficulties. When the time delay is large, the control law based on the angular momentum conservation needs some modifications. How to compensate for such delay, and what kind of effects such delay causes on the parameter estimation results is also discussed. These effects are analyzed by the method of averaging. The asymptotic behavior of the estimated parameters is clarified through the analysis. Numerical simulations are executed to verify the analytical results.

## Model of a Satellite with Mobile Bodies

Here we consider an artificial satellite that has mobile bodies (e.g., antennas or manipulators) and that uses reaction wheels for the attitude control. It is assumed that no external forces such as thrusters are exerted on the satellite during the mobile bodies movement and that the angular momentum around the center of mass of the whole satellite is conserved at zero. Then the following equation holds[5]:

$$M(\phi)\dot{\theta} + f(\phi, \dot{\phi}) + h = 0 \tag{1}$$

The left-hand side of Eq. (1) represents the angular momentum around the center of mass of the whole satellite in the body-fixed coordinates. The mass matrix $M$ depends only on $\phi$ in these coordinates.

We will make the following assumptions for the angles $\phi$ and the rates $\dot{\phi}$ of mobile bodies: 1) $\phi$ and $\dot{\phi}$ are realized according to the commanded values and 2) $\phi$, $\dot{\phi}$, and $\ddot{\phi}$ are all bounded. Mobile bodies are usually driven by stepping motors[3,4] or servomotors with strong local feedback.[7] In both cases, the dynamics of $\phi$ and $\dot{\phi}$ are much faster than the attitude dynamics and can be neglected. In other words, $\phi$ and $\dot{\phi}$ are considered the same as the commanded values. Also mobile bodies are usually driven smoothly. Then $\phi$, $\dot{\phi}$, and $\ddot{\phi}$ are all bounded.

The term $f(\phi, \dot{\phi})$ in Eq. (1) can be separated into[12]

$$f(\phi, \dot{\phi}) = A(\phi, \dot{\phi})\alpha \tag{2}$$

where $\alpha$ is a constant vector, which consists of mass and moments of inertia of the mobile bodies. In this paper, this vector $\alpha$ is regarded as the vector of unknown parameters and $A(\phi, \dot{\phi})$ is regarded as a known matrix dependent on $\phi$ and $\dot{\phi}$. A concrete form of $\alpha$ and $A(\phi, \dot{\phi})$ will appear in the later example. The distinctive feature of Eq. (2) is that $f(\phi, \dot{\phi})$ is linear with respect to the unknown parameters $\alpha$. An adaptive control law will be derived through this property.

## Estimation Law for the Unknown Parameters

Suppose that angular momentum of the wheels $h$ coincides with the commanded value $h_c$. In practice, there is a time delay between $h$ and $h_c$ because of the wheel velocity loop. In this section, the effects of this delay are ignored to give a framework of the adaptive control law. Under this condition, we consider the following control law based on the adaptive control law for the ground-based manipulators[11]:

$$\dot{\hat{\alpha}} = -PA^T(\phi, \dot{\phi})\theta \tag{3}$$

$$h = -A(\phi, \dot{\phi})\hat{\alpha} + K_P\theta + K_I\xi, \qquad \xi = \int_0^t \theta \, \mathrm{d}t \tag{4}$$

Equation (3) is an estimation law for the unknown parameters, whereas Eq. (4) is an attitude control law using the estimated parameters. Equation (4) expresses the conventional feedforward control law when the values of $\hat{\alpha}$ are fixed. Substituting Eqs. (2) and (4) into Eq. (1), we obtain

$$M(\phi)\dot{\theta} + A(\phi, \dot{\phi})(\alpha - \hat{\alpha}) + K_P\theta + K_I\xi = 0 \tag{5}$$

Therefore, if the estimated values $\hat{\alpha}$ are equal to $\alpha$, then the angular momentum of the mobile bodies is canceled.

The stability of Eqs. (3) and (4) can be checked by a Lyapunov function candidate $V$ defined as

$$V = \tfrac{1}{2}\left\{\theta^T M(\phi)\theta + \xi^T K_I \xi + (\alpha - \hat{\alpha})^T P^{-1}(\alpha - \hat{\alpha})\right\} \tag{6}$$

Differentiating $V$ by time along the trajectory of Eqs. (1), (3), and (4), we obtain

$$\dot{V} = -\theta^T\left[K_P - \tfrac{1}{2}\dot{M}(\phi)\right]\theta \tag{7}$$

Hence, so long as the condition

$$K_P > \frac{1}{2}\dot{M}(\phi) = \frac{1}{2}\sum_{i=1}^{n}\frac{\partial M(\phi)}{\partial \phi_i}\dot{\phi}_i \tag{8}$$

is satisfied throughout the trajectory, $\dot{V} \le 0$ holds and $\theta \to 0$ by Barbalat's lemma[13] (details are given in the Appendix), which guarantees the stability of the attitude control system. As $\phi$ and $\dot{\phi}$ are assumed to be realized according to the commanded values, the condition (8) is satisfied if the control gain $K_P$ is fairly large. This is not a severe constraint on the controller design.

## Effects of the Delay in the Wheel Loop

### Asymptotic Behavior of Estimated Parameters $\hat{\alpha}$

Equations (3) and (4) are fundamental forms of the adaptive control law for an artificial satellite with mobile bodies. But they are impractical because they do not consider the effects of the wheel velocity loop. Considering these effects, we modify the adaptive control law and examine its stability under a few assumptions.

The wheel velocity loop is typically represented by Fig. 2, where $K_w$ is a control gain, $1/s$ is wheel dynamics ($s$ is the Laplace transformation parameter). Thus, the transfer function from $h_c$ to $h$ becomes a first-order lag system as

$$\tilde{h} = \frac{1}{1 + \tau_w s}\tilde{h}_c \tag{9}$$

where $\tau_w$ $(=1/K_w)$ is the time constant of the wheel velocity loop and the tilde indicates the Laplace transformation operator.

When a delay exists between $h_c$ and $h$, it is impossible to compensate the angular momentum of the mobile bodies completely even if the unknown parameters are fully estimated. To alleviate the effects of such delay, we propose to add a phase lead to the angular momentum compensation term $A(\phi, \dot{\phi})\alpha$. Thus, the proposed control law becomes

$$\tilde{h}_c = -\frac{1 + \tau_w s}{1 + \tau_f s}\tilde{\hat{f}}(\phi, \dot{\phi}) + K_P\tilde{\theta} + K_I\frac{\tilde{\theta}}{s} \qquad (\tau_f < \tau_w)$$
$$\hat{f}(\phi, \dot{\phi}) = A(\phi, \dot{\phi})\hat{\alpha} \tag{10}$$

The first term on the right-hand side of Eq. (10) represents a phase lead term to compensate for the delay in the wheel velocity loop. The block diagram of the control system composed of Eqs. (3) and (10) is shown in Fig. 3.

Substituting Eq. (10) into Eq. (9), we obtain

$$\tilde{h} = -\frac{1}{1 + \tau_f s}\tilde{\hat{f}}(\phi, \dot{\phi}) + \frac{1}{1 + \tau_w s}\left(K_P\tilde{\theta} + K_I\frac{\tilde{\theta}}{s}\right) \tag{11}$$

Expressing Eq. (11) in the time domain, we obtain

$$h = h_1 + h_2 + h_3$$
$$\dot{h}_1 = -(1/\tau_f)\{h_1 + \hat{f}(\phi, \dot{\phi})\}$$
$$\dot{h}_2 = -(1/\tau_w)(h_2 - K_P\theta)$$
$$\dot{h}_3 = -(1/\tau_w)(h_3 - K_I\xi) \tag{12}$$

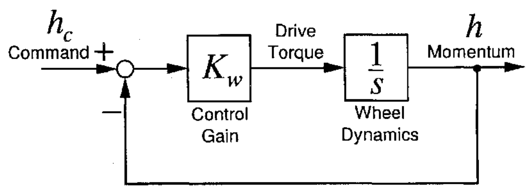


**Fig. 2 Wheel velocity loop.**

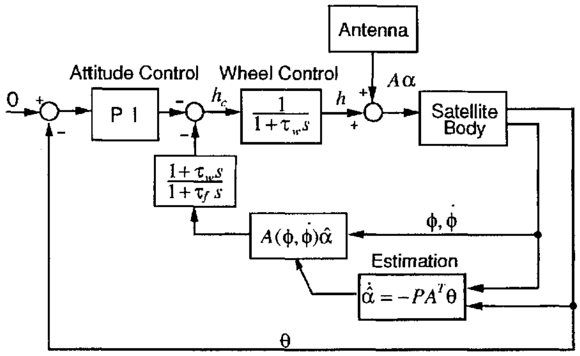


**Fig. 3 Block diagram of control system.**

Now we examine the behavior of the system combined with the control law in Eq. (12) and the estimation law in Eq. (3). For this purpose, we consider the case where the following assumptions hold: 1) movement of the mobile bodies is periodic, 2) dynamics of Eq. (3) are much slower than that of the mobile bodies, and 3) the part of $M(\phi)$ dependent on $\phi$ is much smaller than that independent of $\phi$.

Assumption 1 means that the movement is in a steady state and that the Fourier transformation is applicable. Although this condition does not always hold during the mission execution, it is realistic to set up a calibration phase where the mobile bodies are driven periodically to estimate unknown parameters. The proposed method especially plays an important role in the phase. We will focus on such a case in the analysis. Assumptions 2 and 3 are conditions to regard $\hat{\alpha}$ and $M(\phi)$ approximately constant. If the estimation law (3) does not considerably change the property of the usual attitude control system, assumption 2 is necessarily satisfied. Assumption 3 is also used for the first approximation in the analysis. From numerical experiences, this assumption is valid in cases where the $M(\phi)$ component dependent on $\phi$ is comparatively large (less than 10% of the component). This condition holds for ordinary mobile bodies such as antennas and manipulators.

In the following, we examine the behavior by the method of averaging.[14] The purpose of the method of averaging is to derive the approximate solution of $\hat{\alpha}$ from the averaged system. To obtain the averaged system, we first introduce the Fourier transformation as commonly used in the method of averaging. A function $f(t)$, which is smooth in $0 \le t \le T_0$, can be expressed by applying the inverse Fourier transformation to the result of the Fourier transformation:

$$f(t) = \sum_n f_n \psi_n, \qquad \sum_n = \lim_{N\to\infty} \sum_{n=-N}^{N}$$

$$\psi_n = (1/\sqrt{T_0}) e^{-j\omega_n t}, \qquad \omega_n = (2\pi n)/T_0$$

$$f_n = \int_0^{T_0} f(t)\psi_n^* \,dt$$

where the asterisk indicates the complex conjugate. Time $T_0$ can be set as the motion period of the mobile bodies in the analysis.

Based on the assumptions, $\theta$ and $A(\phi, \dot{\phi})$ are expressed as

$$\theta = \sum_n \theta_n \psi_n, \qquad A(\phi, \dot{\phi}) = \sum_n A_n \psi_n \tag{13}$$

Then the terms $f(\phi, \dot{\phi})$ and $\hat{f}(\phi, \dot{\phi})$ are expressed as

$$f(\phi, \dot{\phi}) = \sum_n A_n \alpha \psi_n, \qquad \hat{f}(\phi, \dot{\phi}) = \sum_n A_n \hat{\alpha} \psi_n \tag{14}$$

Similarly $h_1$, $h_2$, and $h_3$ of Eq. (12) are expressed as

$$\begin{aligned} h_1 &= -\sum_n \frac{1}{1 + j\omega_n \tau_f} A_n \hat{\alpha} \psi_n \\ h_2 &= \sum_n \frac{1}{1 + j\omega_n \tau_w} K_P \theta_n \psi_n \\ h_3 &= \sum_n \frac{1}{(1 + j\omega_n \tau_w) j\omega_n} K_I \theta_n \psi_n \end{aligned} \tag{15}$$

Substituting Eqs. (14) and (15) into Eq. (1), we obtain $\theta_n$ from Eq. (13) as

$$\theta_n = \frac{1}{1 + j\omega_n \tau_f} Y_n A_n \hat{\alpha} - Y_n A_n \alpha \tag{16}$$

where

$$Y_n = \left[ j\omega_n M(\phi_m) + \frac{1}{1 + j\omega_n \tau_w} K_P + \frac{1}{(1 + j\omega_n \tau_w) j\omega_n} K_I \right]^{-1}$$

and $\phi_m$ are the average values of $\phi$, and the following identity is used:

$$\int_0^{T_0} \psi_n^* \psi_m \,dt = \begin{cases} 1: & n = m \\ 0: & n \ne m \end{cases} \tag{17}$$

Now we examine the behavior of $\hat{\alpha}$. From assumption 2, the dynamics of $\hat{\alpha}$ are slow and its asymptotic behavior can be examined by the averaged system, namely,

$$\dot{\hat{\alpha}} \approx -\frac{1}{T_0} \int_0^{T_0} P A^T(\phi, \dot{\phi}) \theta \,dt \tag{18}$$

The procedure of Eq. (18) is referred to as the principle of averaging.[14] Substituting Eqs. (13) and (16) into Eq. (18) and exchanging the order of the integration and the summations, we obtain

$$\begin{gathered} \dot{\hat{\alpha}} = -P(C_f \hat{\alpha} - C_0 \alpha) \\ C_f = \frac{1}{T_0} \sum_n \frac{1}{1 + j\omega_n \tau_f} A_n^* Y_n A_n, \qquad C_0 = \frac{1}{T_0} \sum_n A_n^* Y_n A_n \end{gathered} \tag{19}$$

where the asterisk represents the transposed complex conjugate and the identity (17) is used. If $C_f$ is a full rank matrix through the frequency richness of $A_n$, then the stability condition of the parameter estimation is that the real parts of the eigenvalues of $PC_f$ are all positive. In this case, $\hat{\alpha}$ has a stable equilibrium point $\hat{\alpha}_\infty$ as

$$\hat{\alpha}_\infty = C_f^{-1} C_0 \alpha \tag{20}$$

Equation (20) shows that the estimated values $\hat{\alpha}$ differ from $\alpha$ because of the time delay $\tau_f$, even if the estimation procedure is stable.

Next we consider the case when the input signal is not sufficiently rich and $PC_f$ and $PC_0$ are not full rank matrices. Matrix $PC_f$ is supposed to have the eigenvalue decomposition as

$$PC_f = T \Lambda T^{-1} \tag{21}$$

where $\Lambda$ is a diagonal matrix whose components are the eigenvalues of $PC_f$ and $T$ is a matrix whose columns are the corresponding eigenvectors. Substituting Eq. (21) into Eq. (19) and multiplying $T^{-1}$ from the left, we obtain

$$\dot{\hat{\beta}} = -\left(\Lambda \hat{\beta} - T^{-1} P C_0 \alpha\right), \qquad \hat{\beta} = T^{-1} \hat{\alpha} \tag{22}$$

Letting $\Lambda_1$ be a diagonal matrix composed of nonzero eigenvalues, we separate $\Lambda$ and partition $\hat{\beta}$ and $T^{-1}$ correspondingly as

$$\Lambda = \begin{bmatrix} \Lambda_1 & \\ & \mathbf{0} \end{bmatrix}, \qquad \hat{\beta} = \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix}, \qquad T^{-1} = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}$$

where the number of rows of $\hat{\beta}_1$ and $S_1$ is equal to that of $\Lambda_1$.

For the case when the nonzero eigenvalues of $PC_f$ are positive and the null space of $(PC_f)^T$ coincides with that of $(PC_0)^T$, i.e.,

$$S_2 P C_0 = \mathbf{0} \tag{23}$$

the vector $\hat{\beta}_2$ maintains its initial values and the vector $\hat{\beta}_1$ converges. From the numerical calculations, Eq. (23) holds approximately if $\tau_f$

is small. In this case, the stationary values of $\hat{\alpha}$ depend on their initial values $\hat{\alpha}_0$. The concrete form of the stationary values $\hat{\alpha}_\infty$ is

$$\hat{\alpha}_\infty = T\begin{bmatrix} \Lambda_1^{-1}S_1PC_0\alpha \\ S_2\hat{\alpha}_0 \end{bmatrix} \tag{24}$$

The results of this subsection are summarized as follows. Under assumptions 1–3, the asymptotic behavior of the estimated parameters $\hat{\alpha}$ is approximated by Eq. (19) and the stability condition of the estimation law is derived from this linear equation. The stationary values of $\hat{\alpha}$ are calculated by Eq. (20) (if $C_f$ is full rank) or Eq. (24) (otherwise).

**Setting of Estimation Gain $P$**

A method to set the estimation gain $P$ is obtained by Eq. (19) when the antenna drive pattern is given. Here we set the estimation gain $P$ such that the estimated parameters $\hat{\alpha}$ converge at the same time constant $\tau_c$.

Suppose that the singular value decomposition of $C_f$ is expressed as

$$C_f = U\Sigma V^T \tag{25}$$

where $U$ and $V$ are orthogonal matrices and $\Sigma$ is a diagonal matrix whose components are singular values of $C_f$. If $C_f$ is not a full rank matrix, then $\Sigma$ is separated into a nonzero singular values part ($\Sigma_1$) and a zero singular values part, and the matrices $U$ and $V$ are partitioned correspondingly as

$$\Sigma = \begin{bmatrix} \Sigma_1 & \\ & 0 \end{bmatrix}, \qquad U = [U_1 \quad U_2], \qquad V = [V_1 \quad V_2]$$

where the number of columns of $U_1$ and $V_1$ is equal to that of $\Sigma_1$. Letting $\epsilon$ be a small positive constant, we will set the estimation gain $P$ as

$$P = (1/\tau_c)\left[U_1\Sigma_1^{-1}U_1^T + (1/\epsilon)U_2U_2^T\right] \tag{26}$$

The matrix $P$ becomes positive definite and the following relation holds:

$$PC_f = (1/\tau_c)U_1V_1^T \tag{27}$$

If $C_f$ is a symmetric positive-semidefinite matrix, $U_1 = V_1$ and $PC_f = (1/\tau_c)U_1U_1^T$. Then the estimated parameters $U_1^T\hat{\alpha}$ converge at the same time constant $\tau_c$ from Eq. (19). Although the matrix $C_f$ is not necessarily symmetric positive semidefinite, a symmetric positive-semidefinite matrix is obtained from the real part of $Y_n$ of Eq. (16). Thus, the similar property holds when the real part of $Y_n$ ($K_P$ term) is superior. This condition is usually satisfied from condition (8).

## Numerical Examples

Here we show the results of numerical simulations using the given control law. The simulation model is shown in Fig. 4. In this model, a two-degree-of-freedom antenna represents the mobile bodies ($n = 2$). The term $f(\phi, \dot{\phi})$ in Eq. (1) is expressed as follows:

$$\begin{aligned} f_x &= -\left\{\mu r_3(r_{1z}+r_2)c_1s_2 + \left(\mu r_3^2 + i_{ax} - i_{az}\right)c_1s_2c_2\right\}\dot{\phi}_1 \\ &\quad - \left\{\mu r_3(r_{1z}+r_2)s_1c_2 + \mu r_3 r_{1y}s_2 + \left(\mu r_3^2 + i_{ay}\right)s_1\right\}\dot{\phi}_2 \\ f_y &= -\left\{\mu r_3(r_{1z}+r_2)s_1s_2 + \left(\mu r_3^2 + i_{ax} - i_{az}\right)s_1s_2c_2\right\}\dot{\phi}_1 \\ &\quad + \left\{\mu r_3(r_{1z}+r_2)c_1c_2 + \mu r_3 r_{1x}s_2 + \left(\mu r_3^2 + i_{ay}\right)c_1\right\}\dot{\phi}_2 \\ f_z &= \left\{\mu r_3 r_{1x}c_1s_2 + \mu r_3 r_{1y}s_1s_2 + \left(\mu r_3^2 + i_{ax}\right)s_2^2 + i_{az}c_2^2\right\}\dot{\phi}_1 \\ &\quad + (\mu r_3 r_{1x}s_1c_2 - \mu r_3 r_{1y}c_1c_2)\dot{\phi}_2 \end{aligned} \tag{28}$$

$$\mu = m_am_b/(m_a+m_b), \qquad c_i = \cos\phi_i, \qquad s_i = \sin\phi_i \quad (i = 1, 2)$$

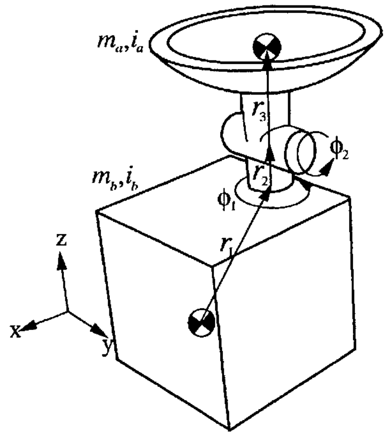


**Fig. 4 Simulation model.**

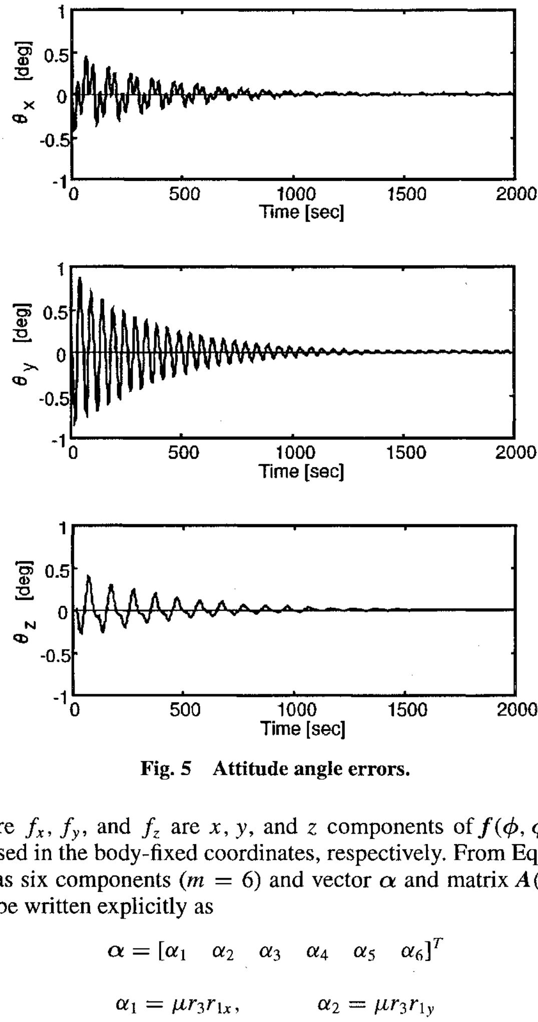


**Fig. 5 Attitude angle errors.**

where $f_x$, $f_y$, and $f_z$ are $x$, $y$, and $z$ components of $f(\phi, \dot{\phi})$ expressed in the body-fixed coordinates, respectively. From Eq. (28), $\alpha$ has six components ($m = 6$) and vector $\alpha$ and matrix $A(\phi, \dot{\phi})$ can be written explicitly as

$$\begin{aligned} &\alpha = [\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \alpha_4 \quad \alpha_5 \quad \alpha_6]^T \\ &\alpha_1 = \mu r_3 r_{1x}, \qquad \alpha_2 = \mu r_3 r_{1y} \\ &\alpha_3 = \mu r_3(r_{1z}+r_2), \qquad \alpha_4 = \mu r_3^2 + i_{ax} \\ &\alpha_5 = \mu r_3^2 + i_{ay}, \qquad \alpha_6 = i_{az} \end{aligned} \tag{29}$$

$$A^T(\phi, \dot{\phi}) = \begin{bmatrix} 0 & s_2\dot{\phi}_2 & a_{31} \\ -s_2\dot{\phi}_2 & 0 & a_{32} \\ -a_{31} & -a_{32} & 0 \\ -c_1s_2c_2\dot{\phi}_1 & -s_1s_2c_2\dot{\phi}_1 & s_2^2\dot{\phi}_1 \\ -s_1\dot{\phi}_2 & c_1\dot{\phi}_2 & 0 \\ c_1s_2c_2\dot{\phi}_1 & s_1s_2c_2\dot{\phi}_1 & c_2^2\dot{\phi}_1 \end{bmatrix} \tag{30}$$

$$a_{31} = c_1s_2\dot{\phi}_1 + s_1c_2\dot{\phi}_2, \qquad a_{32} = s_1s_2\dot{\phi}_1 - c_1c_2\dot{\phi}_2$$

**Table 1 Model parameters**

| Symbol | Unit | Value |
|---|---|---|
| $m_a$ | kg | 200 |
| $m_b$ | kg | 1000 |
| $i_{ax}, i_{ay}, i_{az}$ | $kgm^2$ | 50, 50, 50 |
| $i_{bx}, i_{by}, i_{bz}$ | $kgm^2$ | 6000, 3000, 6000 |
| $r_{1x}, r_{1y}, r_{1z}$ | m | 0.2, 1, 1 |
| $r_2$ | m | 1 |
| $r_3$ | m | 0.3 |

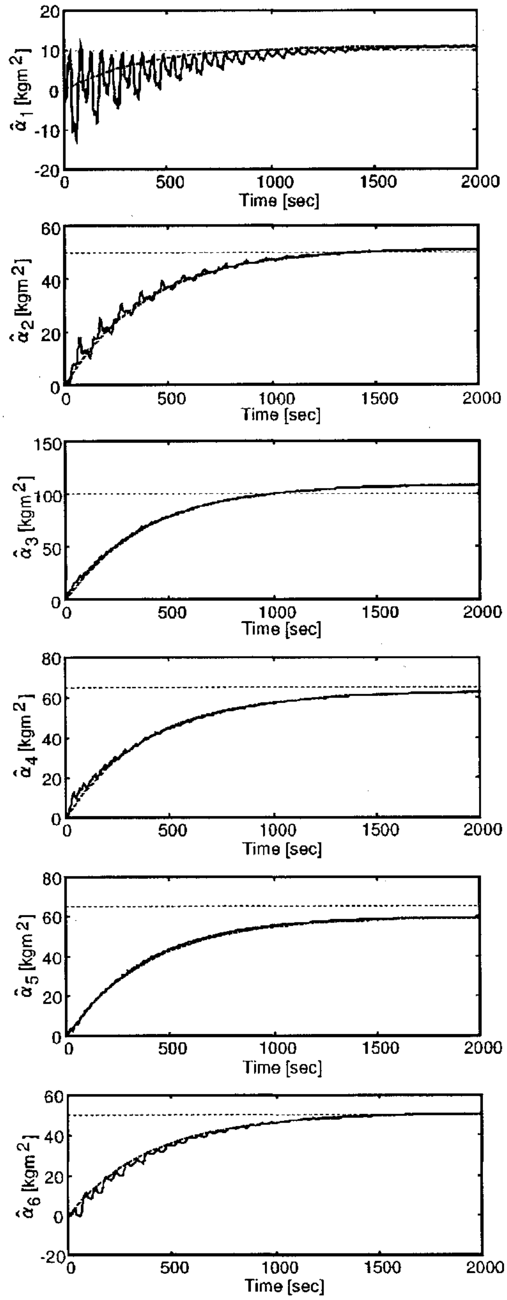

**Fig. 6 Behavior of $\hat{\alpha}$: ····, true value; ——, simulation; and - - - -, analysis.**

The parameter values are shown in Table 1. Then vector $\alpha$ in kilogram square meter becomes

$$\alpha = [10 \quad 50 \quad 100 \quad 65 \quad 65 \quad 50]^T$$

The antenna drive pattern is set as

$$\phi_1 = \phi_{10}\cos[2\pi t/T], \qquad \phi_2 = \phi_{20}\cos[4\pi t/T]$$

$$\phi_{10} = -(\pi/3), \qquad \phi_{20} = -(\pi/6), \qquad T = 100$$

A drive pattern of $0 \sim T$ (s) is regarded as one drive cycle and the simulation repeats this cycle. The input signal has only two sinusoids, but the ranks of $\boldsymbol{C}_f$ and $\boldsymbol{C}_0$ are as many as five due to the nonlinearity in $\boldsymbol{A}(\phi, \dot{\phi})$. Furthermore, condition (23) holds approximately in this case. The attitude control gains in newton meter per radian and newton meter per radian per second, respectively, are set as

$$\boldsymbol{K}_P = \mathrm{diag}(710 \quad 394 \quad 622)$$

$$\boldsymbol{K}_I = \mathrm{diag}(17.7 \quad 9.8 \quad 15.6)$$

These gains are designed so that the control bandwidth is about 0.1 rad/s for each axis. The time delay in the wheel velocity loop is set as $\tau_w = 1.0$ s. The estimation gain $\boldsymbol{P}$ is set from Eq. (26) with $\epsilon = 10^{-4}$. In the following examples, a different parameter $\tau_f$ gives a different estimation gain $\boldsymbol{P}$. The time constant $\tau_c$ is set longer than the time constant of the attitude controller to avoid changing its characteristics a lot. Here the time constant $\tau_c = 400$ s is chosen.

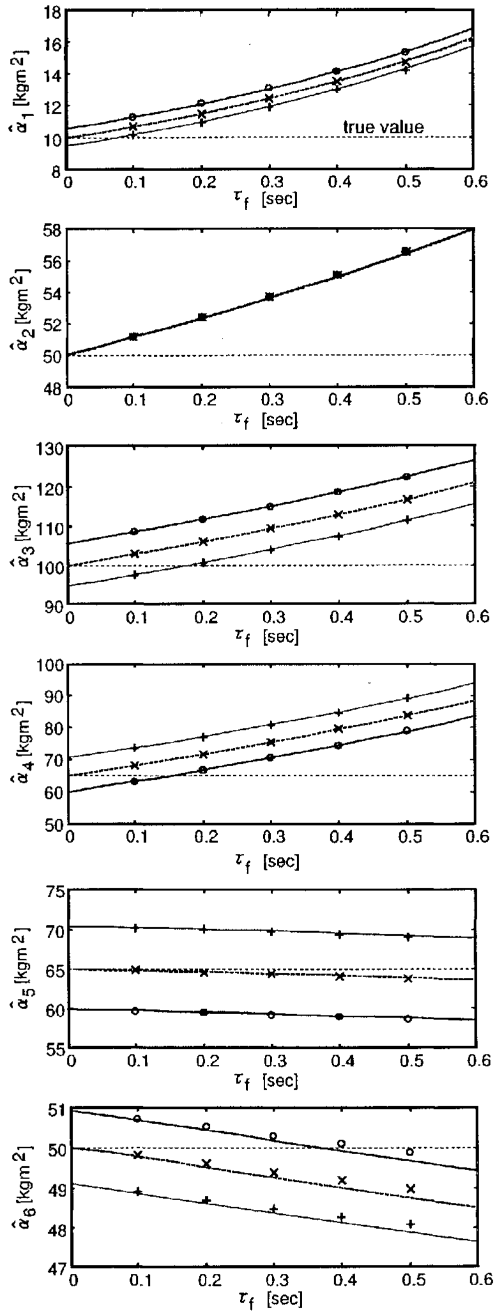


**Fig. 7 Convergent values of $\hat{\alpha}$. $\hat{\alpha}_0 = 0$: ○, simulation ——, analysis; $\hat{\alpha}_0 = \alpha$: ×, simulation - - - -, analysis; and $\hat{\alpha}_0 = 2\alpha$: +, simulation ——, analysis.**

The simulation results of the attitude control are shown in Fig. 5. This figure shows the attitude errors $\boldsymbol{\theta}$ at $t = 0$–2000 s in the case of $\tau_f = 0.1$ s, where the initial values $\hat{\alpha}_0$ are all zeros. In Fig. 5, the attitude errors decrease with time $t$. The reason is that the unknown parameters $\alpha$ are estimated to some extent.

The behavior of the estimated parameters $\hat{\alpha}$ at $t = 0$–2000 s is shown in Fig. 6 as compared with the results of Eq. (19). Figure 6 shows the case of $\tau_f = 0.1$ s where the initial values $\hat{\alpha}_0$ are all zeros. The solid lines show the simulation results of Eqs. (1), (3), and (12), and the dashed lines show the time integration of Eq. (19). The dotted lines show the true values of $\hat{\alpha}(\alpha)$. It is clear from Fig. 6 that the values of $\hat{\alpha}$ converge in the vicinity of the true values, and that the approximation of Eq. (19) expresses well the behavior of the estimated parameters $\hat{\alpha}$.

Figure 7 shows the effects of the initial values $\hat{\alpha}_0$ and the time delay $\tau_f$ on the convergent values of $\hat{\alpha}$. The convergent values of $\hat{\alpha}$ are compared with the values of Eq. (24) by taking $\tau_f$ as the abscissa, where the initial values $\hat{\alpha}_0$ are set at $\mathbf{0}$, $\alpha$, and $2\alpha$. The values of $\hat{\alpha}$ at $t = 4000$ s in the simulation of Eqs. (1), (3), and (12) are used as the convergent values of $\hat{\alpha}$. The symbols $\circ$, $\times$, and $+$ correspond to the convergent values for $\hat{\alpha}_0 = \mathbf{0}$, $\alpha$, and $2\alpha$, respectively. Also, the thick lines, the dashed lines, and the thin lines correspond to the values of Eq. (24) for $\hat{\alpha}_0 = \mathbf{0}$, $\alpha$, and $2\alpha$, respectively. From Fig. 7, the convergent values of $\hat{\alpha}$ are almost equal to the values estimated by Eq. (24).

## Conclusion

This paper presents feedforward control of the motion of mobile bodies using a parameter estimation law for an artificial satellite with antennas or manipulators. The proposed method utilizes the linearity of the unknown parameters on the angular momentum. It is possible to append estimation capability by adding a few terms to the conventional feedforward control law.

This paper also proposes a compensation method for the time delay in the velocity control loop of the wheels and analyzes its effects on the parameter estimation results by the method of averaging. The stability condition of the estimation law is derived and the method of setting the estimation gain matrix is also obtained. The analytical results coincide well with the simulation results, and the effects of the time delay on the parameter estimation are expressed quantitatively.

## Appendix: Proof of $\boldsymbol{\theta} \to \mathbf{0}$ for the System of Eqs. (1), (3), and (4)

In this Appendix, proof of $\boldsymbol{\theta} \to \mathbf{0}$ is given for the system of Eqs. (1), (3), and (4). Because the angles $\phi$ and rates $\dot{\phi}$ are time functions, this system becomes nonautonomous. For such a system, to prove $\boldsymbol{\theta} \to \mathbf{0}$ as $t \to \infty$ from Barbalat's lemma,[13] it is enough to show that there exists a scalar function $V$ that satisfies the following: 1) $V$ is lower-bounded, 2) $\dot{V}$ is negative semidefinite, and 3) $\dot{V}$ is uniformly continuous in time. The function $V$ in Eq. (6) satisfies 1 and 2 under the condition of Eq. (8). To show $\dot{V}$ is uniformly continuous, it is enough to prove $\ddot{V}$ is bounded. From Eq. (7), the function $\dot{V}$ is rewritten as

$$\dot{V} = -\boldsymbol{\theta}^T\left[\boldsymbol{K}_P - \frac{1}{2}\dot{\boldsymbol{M}}(\phi)\right]\boldsymbol{\theta} = -\boldsymbol{\theta}^T\left(\boldsymbol{K}_P - \frac{1}{2}\sum_{i=1}^{n}\frac{\partial \boldsymbol{M}(\phi)}{\partial \phi_i}\dot{\phi}_i\right)\boldsymbol{\theta} \tag{A1}$$

Thus, $\ddot{V}$ is calculated as

$$\ddot{V} = -2\dot{\boldsymbol{\theta}}^T\boldsymbol{K}_P\boldsymbol{\theta} + \frac{1}{2}\boldsymbol{\theta}^T\left(\sum_{i=1}^{n}\frac{\partial \boldsymbol{M}(\phi)}{\partial \phi_i}\ddot{\phi}_i + \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2 \boldsymbol{M}(\phi)}{\partial \phi_i \partial \phi_j}\dot{\phi}_i\dot{\phi}_j\right)\boldsymbol{\theta} \tag{A2}$$

Since $\dot{V} \leq 0$ under the condition of Eq. (8), $V(t) \leq V(0)$ $(t \geq 0)$. This shows that $\boldsymbol{\theta}, \boldsymbol{\xi}$, and $\hat{\alpha}$ are bounded. As $\phi$, $\dot{\phi}$, and $\ddot{\phi}$ are bounded from the assumption, $\dot{\boldsymbol{\theta}}$ is bounded from Eq. (5) and $\ddot{V}$ becomes bounded from Eq. (A2). Therefore, Barbalat's lemma indicates $\boldsymbol{\theta} \to \mathbf{0}$ as $t \to \infty$.

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